Lijiana

Integriation (Anti-derivative). $\int$

$f'(x)$ given, find $f(x)$.

$\underbrace{\int 2x\,dx}_{f'(x)} = x^2 + C \to$ any constant.

$\int$ sum dx = sum of $\int$

$$\int f_1(x) \pm f_2(x)\,dx = \int f_1(x)\,dx + \int f_2(x)\,dx.$$

$$\int (4x^2 + 6x - 10)\,dx = \frac{4}{3}x^3 + 3x^2 - 10x + C$$

Result: ① $\int \underset{\text{constant}}{a} x^{n} \,dx = \frac{a}{n+1} x^{n+1} + C \quad (n \neq -1)$ *

② $\int \underset{\text{number}}{a}\, u'(x)[u(x)]^{n}\,dx = \frac{a}{n+1}[u(x)]^{n+1} + C \quad n \neq -1$

$\int 4\sqrt[4]{x^3} + \frac{2}{x^{10}}\,dx$. Rewrite: $\int x^{\frac{3}{4}} + 2x^{-10}\,dx$

$$= \frac{1}{\frac{3}{4}+1} x^{\frac{3}{4}+1} + \frac{2}{-10+1} x^{-10+1} + C$$

$$= \frac{1}{\frac{7}{4}} x^{\frac{7}{4}} + \frac{2}{-9} x^{-9} + C$$

$$= \frac{4}{7} x^{\frac{7}{4}} - \frac{2}{9} x^{-9} + C$$

$\int x^3 (x^4 - 1)^{13}\,dx$ Rewrite: ① $u(x) = x^4 - 1$

$u'(x) = 4x^3$

② $\frac{1}{4}\int 4x^3 (x^4-1)^{13}$

③ $= \frac{1}{4} \cdot \frac{1}{13+1} (x^4-1)^{13+1} + C$

$= \frac{1}{4} \cdot \frac{1}{14} (x^4-1)^{14} + C$

$= \frac{1}{56} (x^4-1)^{14} + C$

$\int x\sqrt{x^2+3}\,dx =$ Rewrite: ① $\int x(x^2+3)^{\frac{1}{2}}\,dx$

② $u(x) = x^2 + 3$

$u'(x) = 2x$

③ $\frac{1}{2}\int 2x\,(x^2+3)^{\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\frac{1}{2}+1} (x^2+3)^{\frac{1}{2}+1} + C$

$= \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (x^2+3)^{\frac{3}{2}} + C = \frac{1}{3}(x^2+3)^{\frac{3}{2}} + C$

common mistake
- missing part will be number only *

$\int \frac{x+1}{\sqrt{x^2+2x-3}} \, dx.$ Rewrite: $\int (x+1)(x^2+2x-3)^{-\frac{1}{2}} \, dx.$

$\downarrow$

$u(x) = x^2+2x-3$

$u'(x) = 2x+2$

$\frac{1}{2}\int 2(x+1)\underbrace{(x^2+2x-3)}_{\text{only}}^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{-\frac{1}{2}+1}(x^2+2x-3)^{-\frac{1}{2}+1}$

$= \frac{1}{2} \cdot \frac{1}{\frac{1}{2}}$

$= (x^2+2x-3)^{\frac{1}{2}} + C$

$\int (6x^2+6)\underbrace{[x^3+3x-10]}_{u(x)}^{13} \, dx$

$u'(x) = 3x^2+3$

$2\int \frac{1}{2}\underbrace{(6x^2+6)[x^3+3x-10]}_{\text{only}}^{13}$

$= 2 \cdot \frac{1}{13+1}[x^3+3x-10]^{13+1} \rightarrow$ only $u(x)$

$= 2 \cdot \frac{1}{14}[x^3+3x-10]^{14}$

$= \frac{1}{7}[x^3+3x-10]^{14}.$

$\int \frac{\ln(3x-1)}{3x-1} \, dx.$

$= \frac{1}{3}\int 3 \cdot \frac{1}{3x+1}(\ln(3x+1))^1$

$\downarrow$

$u = \ln(3x+1)$

$u' = \frac{3}{3x+1}$

$= \frac{1}{3} \cdot \frac{1}{2}\ln[3x+1]^2 + C$

$\int a e^{k(x)} \, dx = a e^{k(x)} + C$

$\int 6(x-1) e^{(3x^2-x+2)}$

$u = 3x^2-x+2$

$u' = 6x-1$

$= e^{(3x^2-x+2)} + C.$

$\frac{1}{2}\int 2x e^{(x^2+1)}$

$u = x^2+1$

$u' = 2x$

$= \frac{1}{2}e^{(x^2+1)} + C$

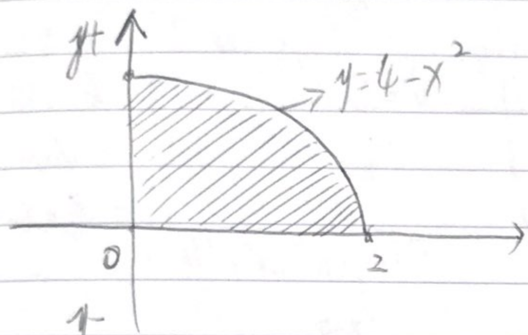
$u' = 4x^3+4$

1). $\frac{1}{4}\int 4(x^3+1)\, e^{(x^4+4x+1)}\, dx = \frac{1}{4} e^{(x^4+4x+1)} + C$

$\frac{1}{1\cdot 2\cdot 2x^{-\frac{1}{2}}}$ $\quad \frac{1}{2}\ \frac{1}{4}$

$x^{\frac{1}{2}}+3 \quad u' = \frac{1}{2}x^{-\frac{1}{2}}$

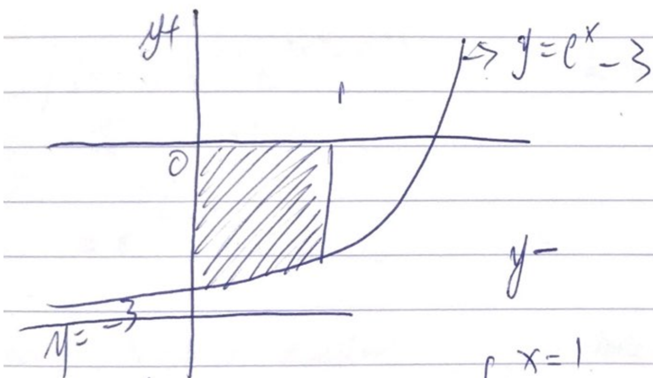
2). $\int \frac{1}{2\sqrt{x}}\, e^{(\sqrt{x}+3)}\, dx = e^{(\sqrt{x}+3)} + C.$



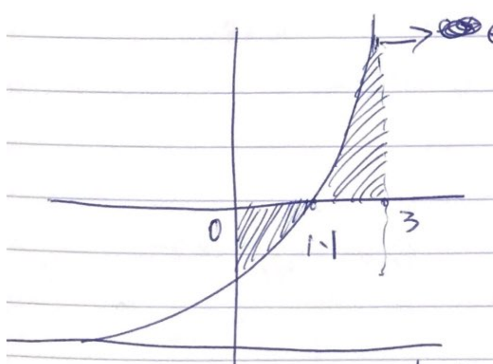
$\int_{x=0}^{x=2} (4-x^2)\,dx = 4x - \frac{1}{3}x^3 \Big|_{x=0}^{x=2} = (8-\frac{1}{3}(8)) = \frac{16}{3}$ units²

1- always minus.

$= 0.$



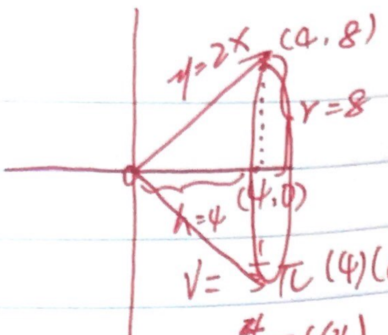
$\int_{x=0}^{x=1} (e^x - 3)\, dx.$



$\left|\int_{x=0}^{x=1.1} (e^x - 3)\,dx\right| =$

$\int_{x=1.1}^{x=3} (e^x - 3)\,dx$

$V = \frac{1}{3}\pi h r^2$



$V = \frac{1}{3}\pi(4)(64)$

$= \frac{4}{3}\pi(64)$

Rotate $y=2x$ about x-axis

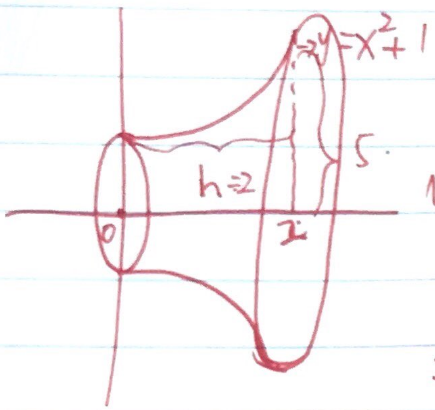
volume (object) $= \int_{x=}^{x=} f(x)^2\,dx$

equation of the curve (y) in terms of x

$\pi\int_{x=0}^{x=4} (2x)^2\,dx$

$= \pi\int_{x=0}^{x=4} 4x^2\,dx = \pi\,\frac{4}{3}x^3$

$= \frac{4}{3}\pi(64)$



$V = \pi\int_{x=0}^{x=2} (x^2+1)^2\,dx$

$= \pi\int_{x=0}^{x=2} (x^4+2x^2+1)\,dx$

$= \cancel{\pi \cdot \frac{4}{5}x^5} = \pi\cdot\frac{1}{5}\cdot x^5 + \frac{2}{3}(x^3) + x$